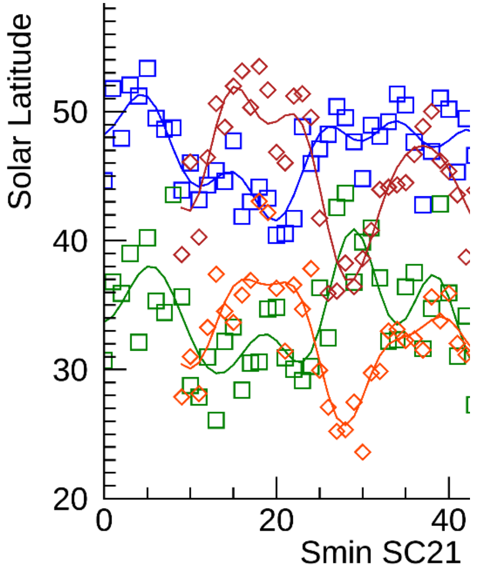
FFT and Wavelet Tools for Harmonic Analyses

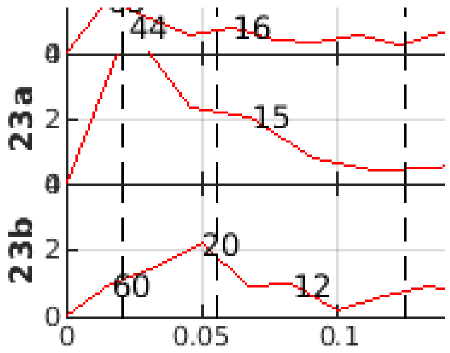


Time series

by Barbara Emery, HAO/NCAR

and IDL or MatLab

and Torrence and Compo (1998)

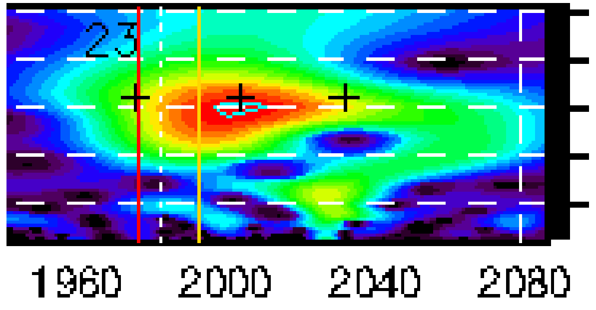


FFT

For WHPI Tools Workshop

Thursday January 21, 2021

Morlet wavelet



‘Noisy Signal’ Example in matlab for fft documentation

Compute the Fourier transform of the signal.

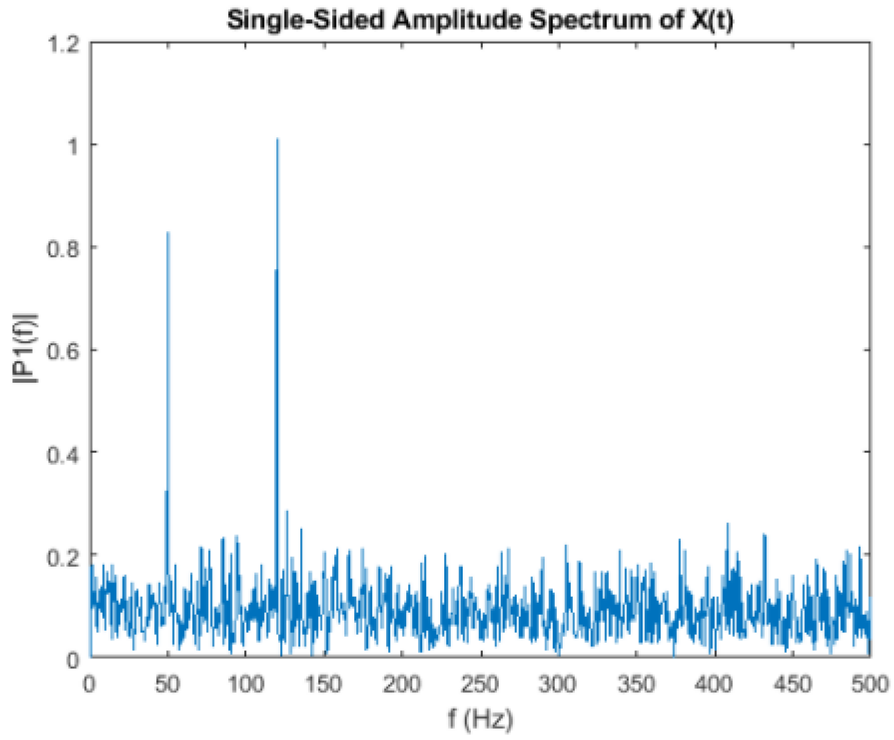
```
Y = fft(X);
```

Compute the two-sided spectrum P2. Then compute the single-sided spectrum P1 based on P2 and the even-valued signal length L.

```
P2 = abs(Y/L);
P1 = P2(1:L/2+1);
P1(2:end-1) = 2*P1(2:end-1);
```

Define the frequency domain f and plot the single-sided amplitude spectrum P1. The amplitudes are not exactly at 0.7 and 1, as expected, because of the added noise. On average, longer signals produce better frequency approximations.

```
f = Fs*(0:(L/2))/L;
plot(f,P1)
title('Single-Sided Amplitude Spectrum of X(t)')
xlabel('f (Hz)')
ylabel('|P1(f)|')
```

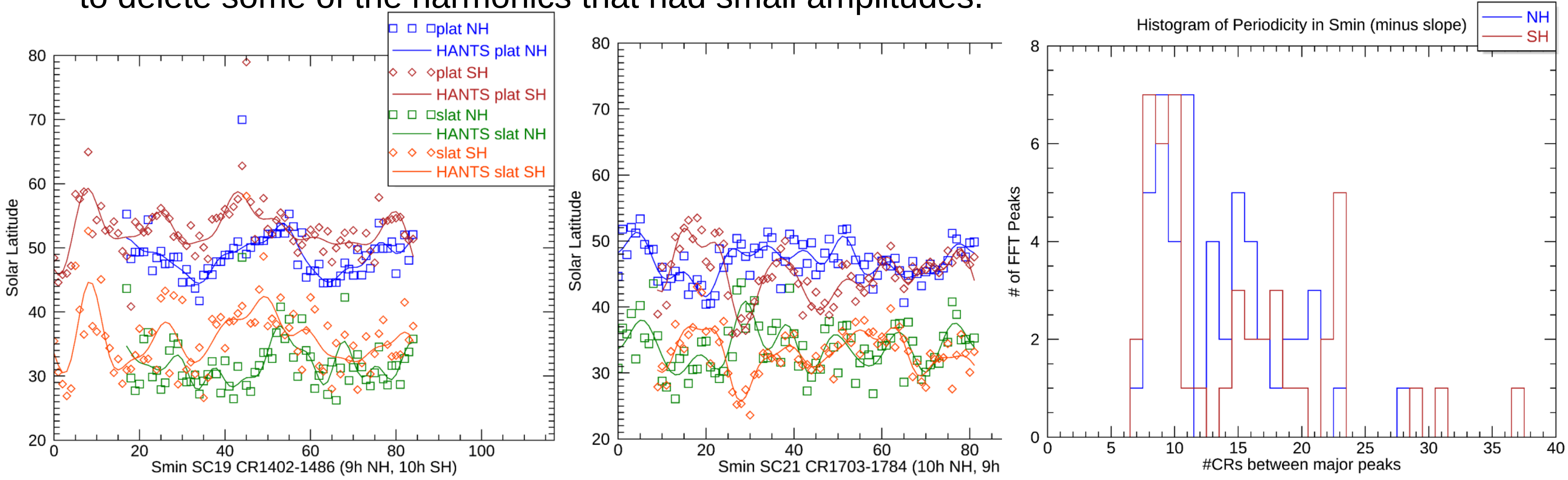


Fast Fourier Transform (FFT)

- Fast Fourier Transform (FFT) is a direct way to find periodicities in data.
- Often have data as a function of evenly spaced time in the geosciences, or can have data as a function of lat/lon (tides etc). Interpolate missing data.
- Assume evenly spaced time Δt. Frequency is inverse of time f=1/t. Because time is evenly spaced, frequency is not (unlike evenly spaced sampling frequency and uneven periods 1/f).
- Highest frequency is Nyquist frequency ½*(1/Δt) with shortest possible period (or resolvable scale) of 2*Δt. Periods increase with multiple powers of 2.
- Longest period is L, the number of points in the sample (lowest f(2)=1/L, except for f(1)=0 where the amplitude is the mean of the sample)
- Number of frequencies is nf=L/2+1 (L even), or nf=(L+1)/2+1 (L odd) where f(1:nf)=(1/Δt)*(0:nf-1)/L so f(1)=0 or zero frequency for the mean
- Find Y Discrete Fourier Transform (DFT) of X with FFT (Y=fft(X(n1:n2), L=n2-n1+1) where X is with any slopes removed for zero baseline
- p2=abs(Y/L), real amplitude is p1=p2, p1(2:nf-1)=2*p1(2:nf-1), sum(amps)=sum[amp(2:nf-1)] (avoid mean amplitude p1(1) at f(1)=0)

Fast Fourier Transform (FFT) from IDL for Lats of PILs (NLs)

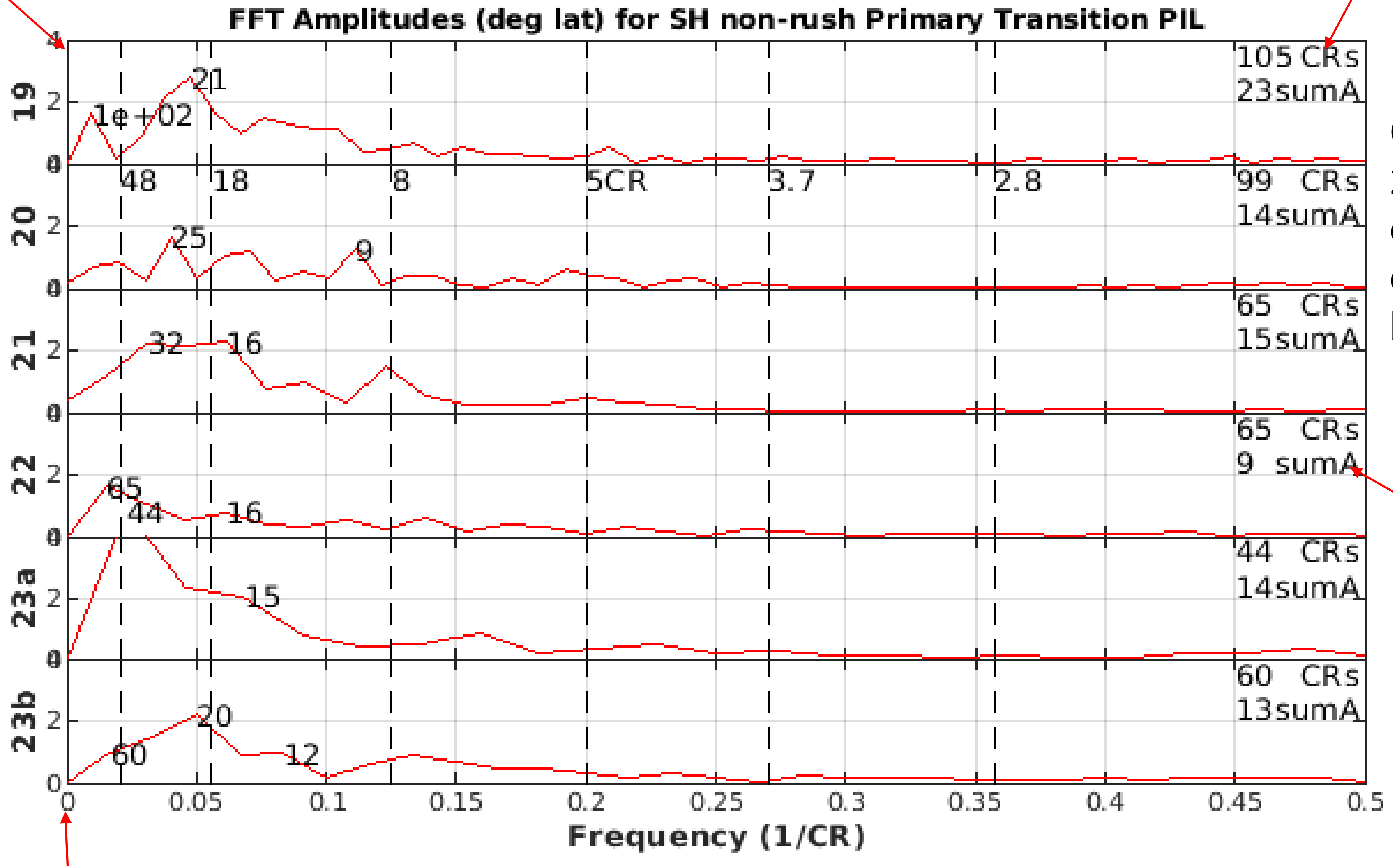
-ts_hants.pro (IDL 8.7 > sswidl) finds amplitudes and phases of several frequencies at once. Tom Kuchar of Boston College got me the code and examples running it. Thanks, Tom!
-ts=Time Series, hants=Harmonic ANalysis of Time Series from NRL creates a time series based on harmonic analysis of time series data using only 4 frequencies in default, which I increased to get a minimum period of ~8CR. Work with zero baseline data (slopes removed).
-The zero harmonic is the mean, the first harmonic is L (number of points in the time series)
-There were some errors in ts_hants.pro depending on if the vector had even or odd # pts (L).
-I also could not figure out how to use the phase to get my own time series estimate if I wanted to delete some of the harmonics that had small amplitudes.



Matlab FFT for Smax to Smin PCF (neutral line closest to pole) SH Latitude Variations

Amplitude p1 in Lat°

L or # pts in FFT sample



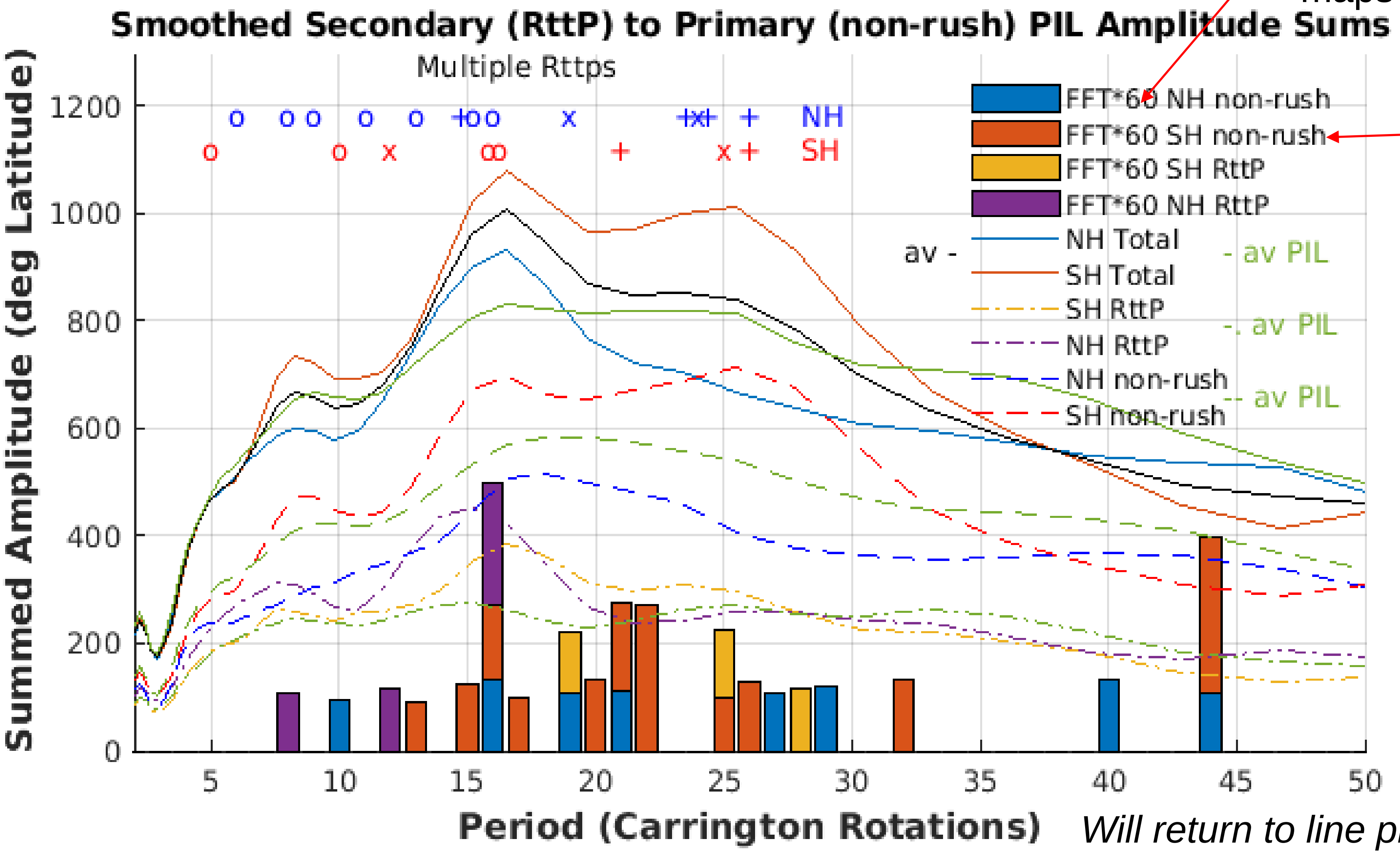
Peaks at L (# pts, 105, 65, 40, 60 CR for SC19, 22, 23a-b) can be real or incomplete removal of slopes for zero baseline values.

Sum(p1(2:nf-1))
p1=FFT Amplitude in Lat°

P1(1)=amplitude of zero frequency f(1), or mean

Bar Plot of FFT summed amplitudes >1.5° as a function of CR (1/freq to nearest integer CR)

11 FFT segments, 5 Rush-to-the-Poles (RttP), 6 non-Rush for 732 Carrington maps or 732/11=66.5 (~60).

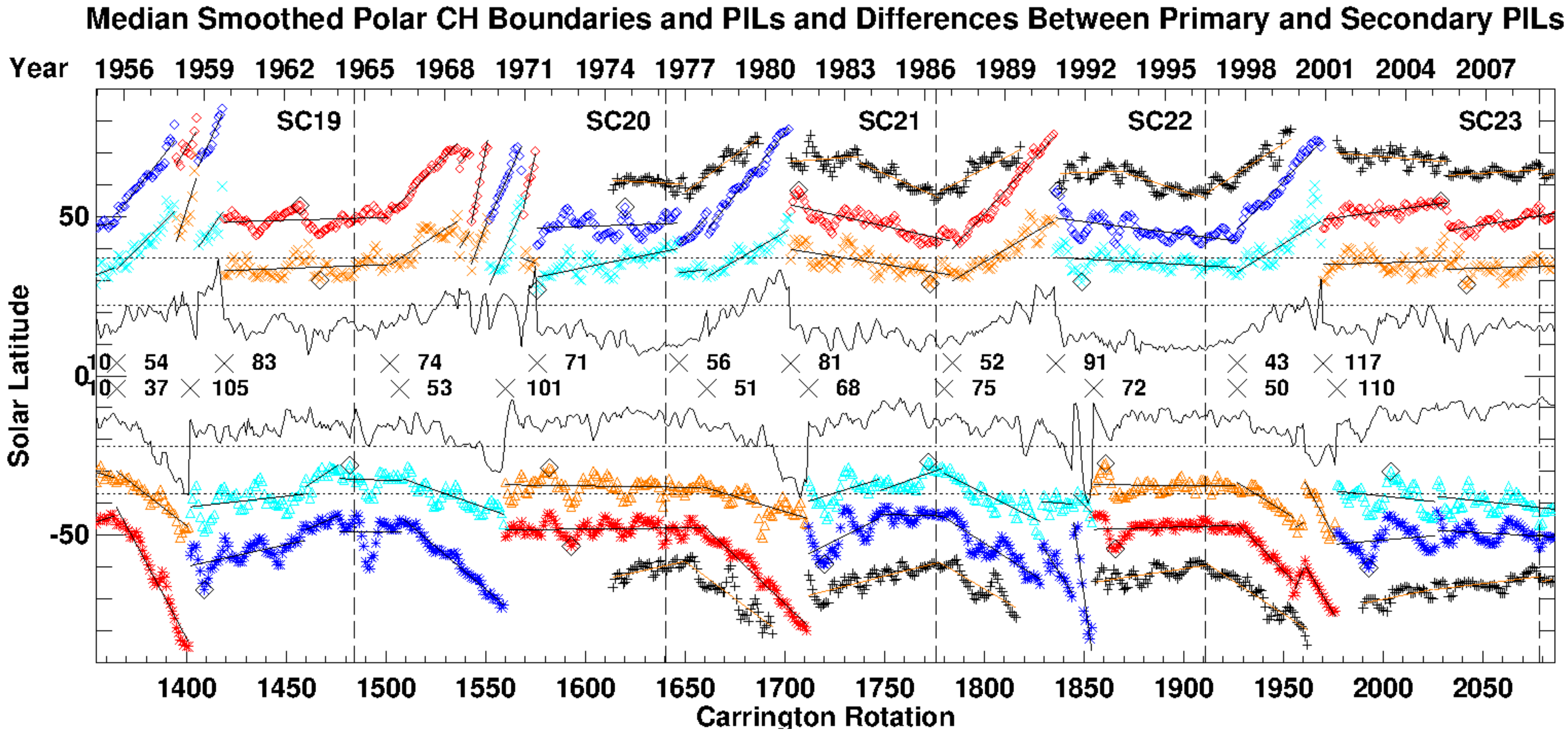


Previous Plot of Amplitudes

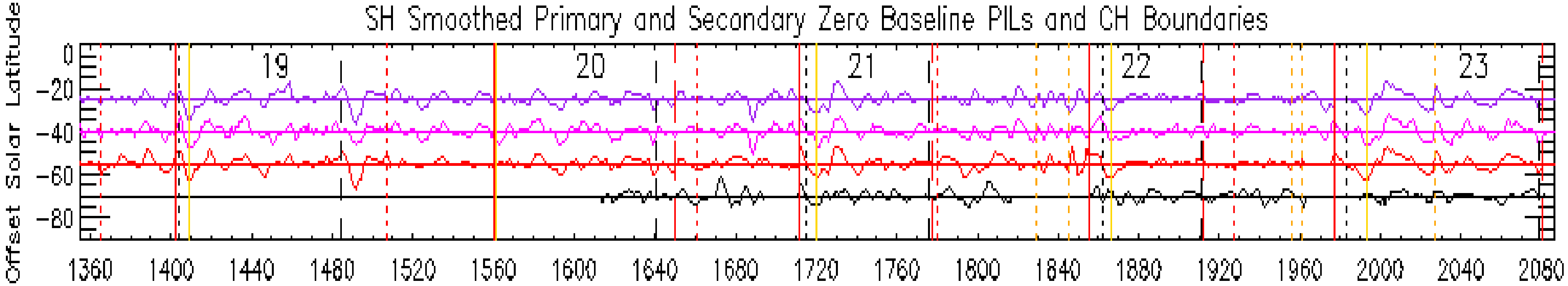
*Should multiply each FFT segment by L, the number of CRs in each before adding the summed amplitudes over 1.5° latitude for this plot. Roughly, 37% of the maps are RttP, so ~54 times 5 RttPs and ~77 times 6 non-Rush and list it as FFT*L instead of FFT*60.*

Will return to line plots for wavelet amplitudes.

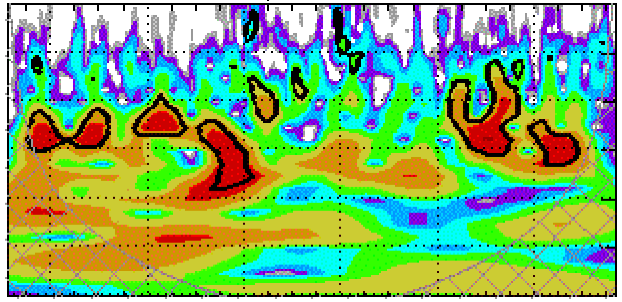
Latitude Location Data to be Analyzed (3-CR Smoothed to reduce noise)



Made with cgplot



Purple = secondary RttP + primary Non-Rush *https://paos.colorado.edu/research/wavelets*



A Practical Guide to Wavelet Analysis

With significance and confidence testing

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Wavelet Analysis & Monte Carlo

References & Web Sites

Interactive Wavelet Plot (no longer available)

Software for Fortran, IDL, Matlab, and Python

Frequently Asked Questions (FAQ)

Article: "A Practical Guide to Wavelet Analysis", C. Torrence and G. P. Compo, 1998*.

- Abstract & List of Topics
- Additional information & Errata
- Google Scholar Citations

Wavelet Coherency and Phase

wavelet.pro
cgloadct.pro
cgplot.pro
cgcontour.pro

omni2_2018304_2020257

plot_psd_amp_Vsw_ap.pro
ampwhpi.pro

I used IDL because I borrowed code from Federico Gasperini of ASTRA. (Thank you SO MUCH, Federico!)

Morlet Wavelets

- Morlet period is similar to the FFT period (=1.03*Morlet period)
- Smallest resolvable scale (period T) is 2Δt (Nyquist frequency f=1/T, like FFT) (s0 eq 9)
- Scales increase as powers of 2 (like FFT)
- Fill in missing data (can be zeroes for zero baseline values, or linearly interpolate first)
- Recommend padding zero baseline array with zeroes to the nearest multiple of 2 (to avoid edge effects at the beginning or the end of the wavelets)
- Zero baseline values remove the mean and significant slopes (like Rush-to-the-Poles)
- Increasing period has increasing intervals between each period (like FFT, multiples of 2) (For Δt=1CR, periods from 2-~66CR had intervals of 0.2CR to 4.9CR or factor of 25)
- Power Spectral Density (PSD) estimates the "true" power underneath the power spectrum curve, but can be "biased" for sharp peaks.
- Amplitude = SQRT[PSD/(2π ΔT)], normalization of 2π from eq 6 for amplitude ~FFT

It is convenient to write the scales as fractional powers of two:

$$s_j = s_0 2^{j\delta j}, \quad j = 0, 1, \ldots, J \qquad (9)$$

$$J = \delta j^{-1} \log_2 (N \delta t / s_0), \qquad (10)$$

where s_0 is the smallest resolvable scale and J determines the largest scale. The s_0 should be chosen so that the equivalent Fourier period (see section 3h) is approximately $2\delta t$. The choice of a sufficiently small δj

c. Normalization

To ensure that the wavelet transforms (4) at each scale s are directly comparable to each other and to the transforms of other time series, the wavelet function at each scale s is normalized to have unit energy:

$$\hat{\psi}(s\omega_k) = \left(\frac{2\pi s}{\delta t} \right)^{1/2} \hat{\psi}_0 (s\omega_k). \qquad (6)$$

~16 CR Oscillations from Solar Maximum Transition Peaks

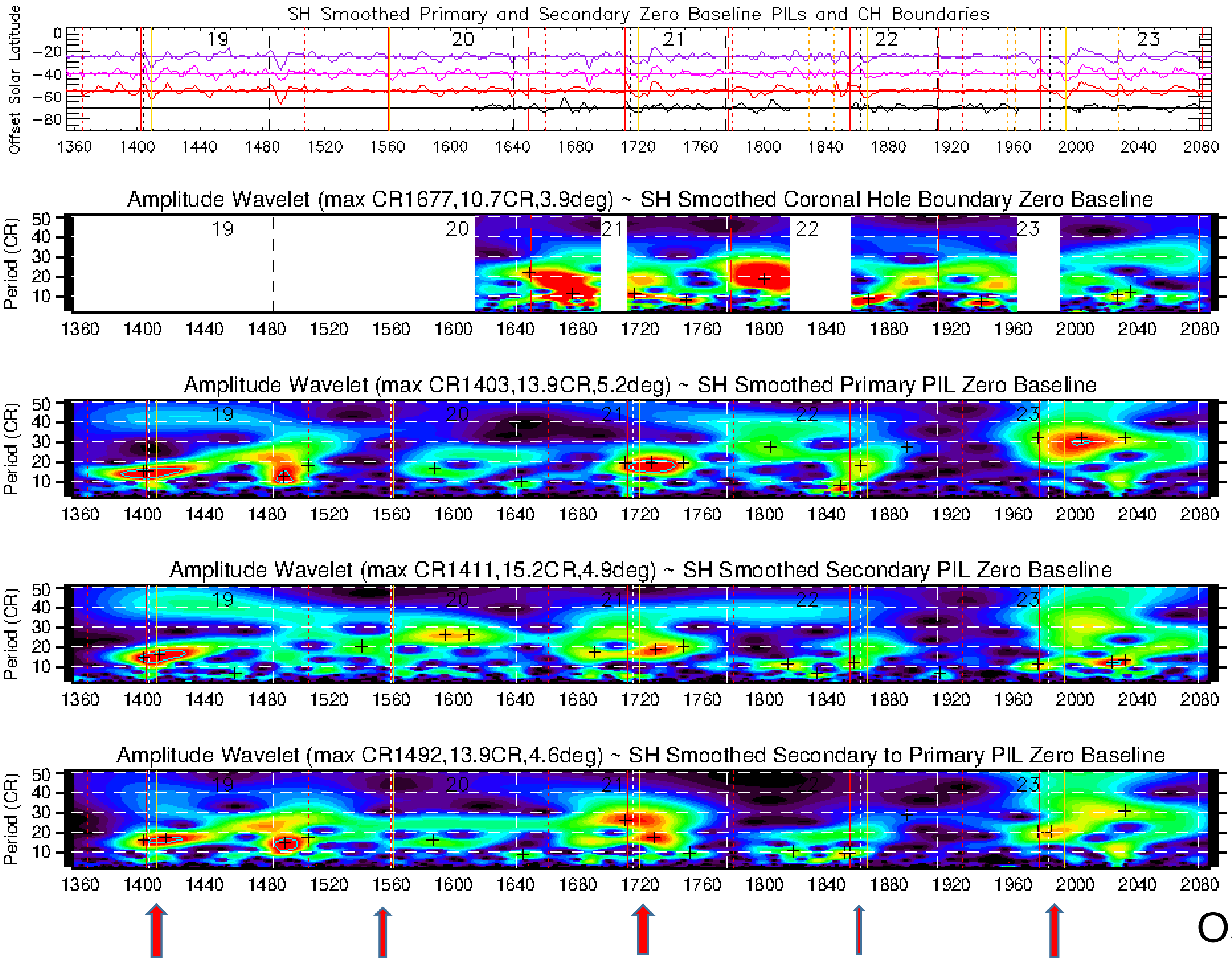
Red lines Smax polarity change when primary PIL ends its RttP

Gold lines transition peaks or secondary PIL RttP endings

SH

Contours made with cgcontour

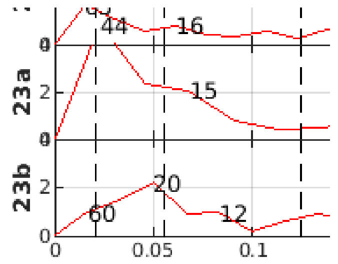
~10-35 CR period of oscillations



Fast Fourier Transform (FFT) Amplitudes (°Lat) and Periods (CR)

SC 23a ~5° ~35CR

SC 23b ~2° ~20+CR



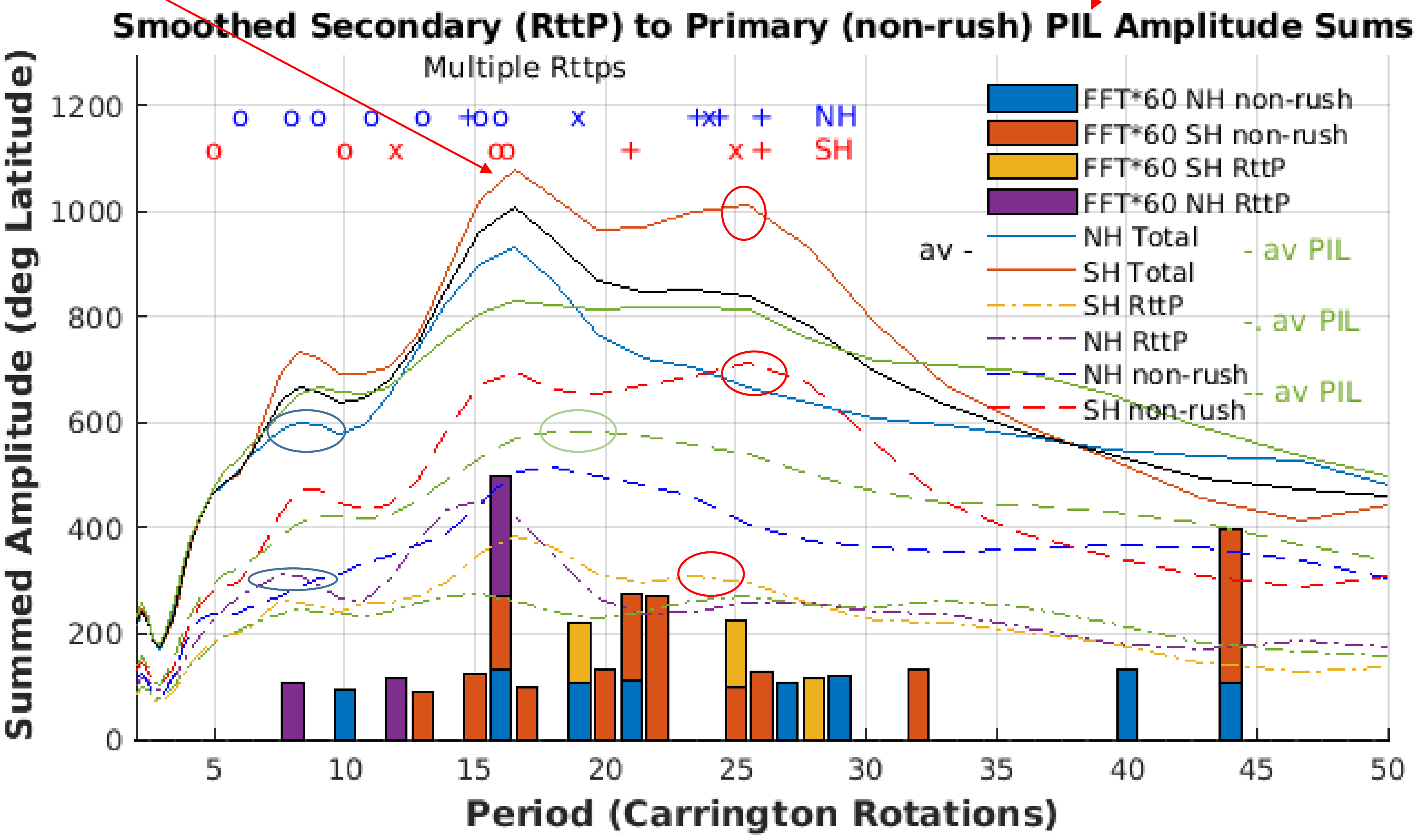
Oscillations ~16 CR Smax Transition Peaks

Largest SH (Southern Hemisphere) oscillations in odd SCs 19, 21, 23

Line Plot of Morlet wavelet summed amplitudes >95% significance level as a function of CR

~16 CR PIL Peak Period from Wavelet Amplitude Sums

Purple curve from previous slide

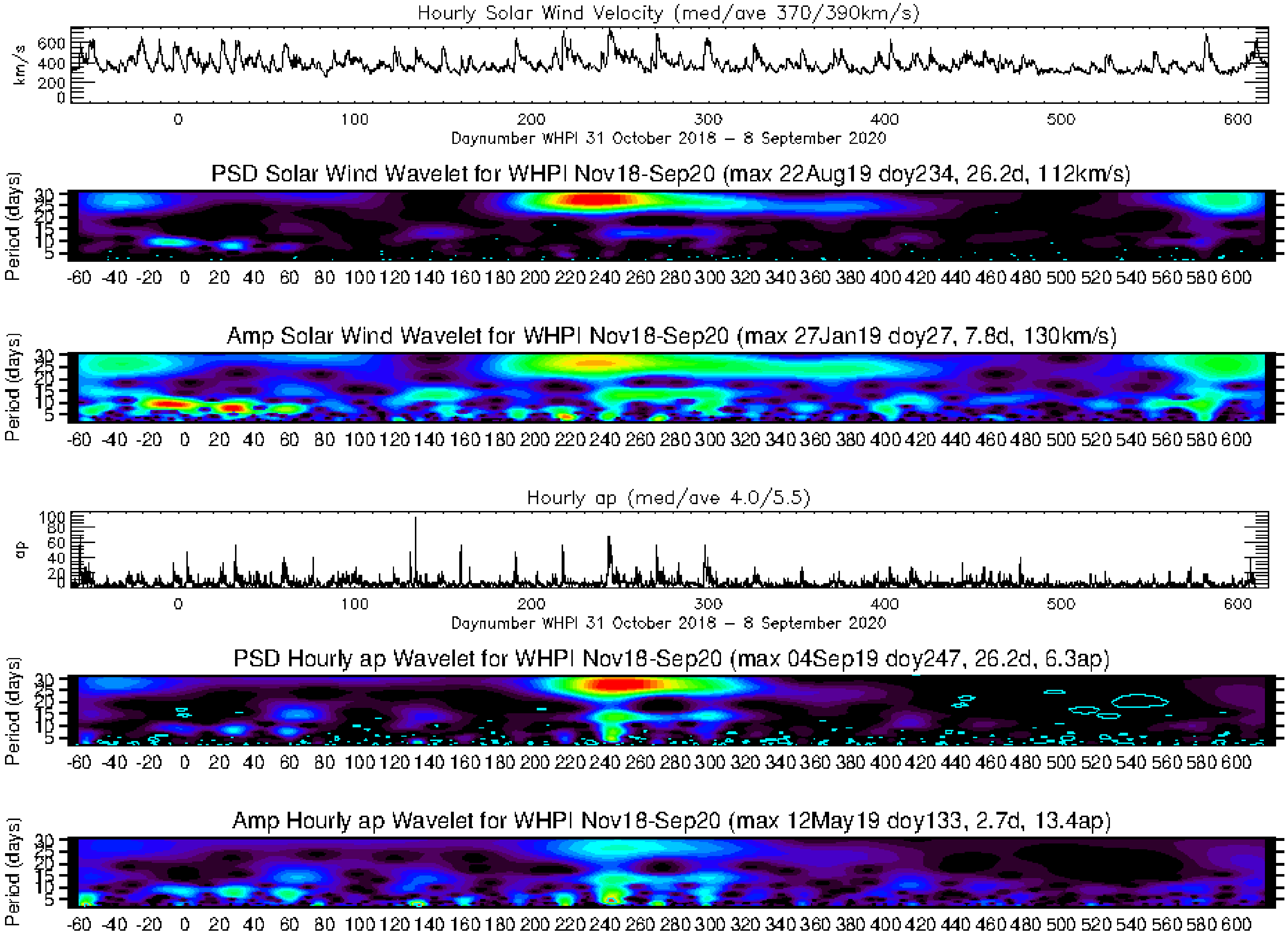


Av PIL = primary + secondary PIL (av NH + SH)

SH peak ~25 CR, non-Rush

NH peak ~8 CR, RttP

Av PIL non-Rush peak ~20 CR



WHPI Examples for Hourly Solar Wind Velocity and Hourly ap magnetic index (linearized Kp)

-Linearly interpolated missing Vsw values (no missing ap)

-Removed median for Vsw, but not for ap

-PSD peaks ~26 days

-Amplitude peaks lower in period because of 1/sqrt(ΔT) (~5x from 2-66CR) 8-day Vsw, 3-day ap